

Nilpotent Symmetries in Jackiw–Pi Model: Augmented Superfield Approach

Saurabh Gupta^(a) and R. Kumar^(b)

^(a)*Instituto de Física, Universidade de São Paulo,
C. Postal 66318, 05314-970 São Paulo, SP, Brazil*

^(b)*S. N. Bose National Centre for Basic Sciences,
Block JD, Sector III, Salt Lake, Kolkata–700098, India*

E-mails: saurabh@if.usp.br; rohit.kumar@bose.res.in

Abstract: We derive the complete set of off-shell nilpotent ($s_{(a)b}^2 = 0$) and absolutely anticommuting ($s_b s_{ab} + s_{ab} s_b = 0$) Becchi-Rouet-Stora-Tyutin (BRST) (s_b) as well as anti-BRST symmetry transformations (s_{ab}) corresponding to the combined Yang-Mills and non-Yang-Mills symmetries of the $(2+1)$ -dimensional Jackiw-Pi model within the framework of augmented superfield formalism. The absolute anticommutativity of the (anti-)BRST symmetries is ensured by the existence of *two* sets of Curci-Ferrari (CF) type of conditions which emerge naturally in this formalism. The presence of CF conditions enables us to derive the coupled but equivalent Lagrangian densities. We also capture the (anti-)BRST invariance of the coupled Lagrangian densities in the superfield formalism. The derivation of the (anti-)BRST transformations of the auxiliary field ρ is one of the key findings which can neither be generated by the nilpotent (anti-)BRST charges nor by the requirements of the nilpotency and/or absolute anticommutativity of the (anti-)BRST transformations. Finally, we provide a bird's-eye view on the role of auxiliary field for various massive models and point out few striking similarities and some glaring differences among them.

PACS numbers: 11.15.-q, 03.70.+k, 11.10Kk, 12.90.+b

Keywords: Jackiw-Pi model; augmented superfield formalism; Curci-Ferrari conditions; (anti-)BRST symmetry transformations; nilpotency and absolute anticommutativity

1 Introduction

The co-existence of mass and gauge invariance *together* is still one of the main issues connected with the gauge theories, in spite of the astonishing success of the standard model of particle physics which is based on (non-)Abelian 1-form gauge theories. However, it is worthwhile to mention that, in the case of sufficiently strong vector couplings, the gauge invariance does not entail the masslessness of gauge particles [1, 2]. Thus, it is needless to say that the mass generation in gauge theories is a crucial issue which has attracted a great deal of interest [3, 4].

In the recent past, many models for the mass generation have been studied in the diverse dimensions of spacetime. In this context, mention can be made of about 4D topologically massive (non-)Abelian gauge theories, with $(B \wedge F)$ term, where 1-form gauge field acquires a mass in a natural fashion [5, 6, 7]. One of the key features associated with such models is that the 1-form gauge field gets a mass without taking any recourse to the Higgs mechanism. We have thoroughly investigated these models within the framework of Becchi-Rouet-Stora-Tyutin (BRST) as well as superfield formalism [8, 9, 10, 11, 12, 13]. It is interesting to point out that the main issues connected with the 4D Abelian topologically massive models are that they suffer from the problems connected with renormalizability when straightforwardly generalized to the non-Abelian case [14]. However, this issue can be circumvented by the introduction of extra field (see, e.g. [15, 16]).

At this juncture, it is worth mentioning about the lower dimensional non-Abelian massive models, such as $(2 + 1)$ -dimensional Jackiw-Pi (JP) model [17], which are free from the above mentioned issues. The silent features of JP model are as follows. First, it is parity conserving model due to the introduction of a 1-form vector field having odd parity. Second, mass and gauge invariance are respected together. Third, it is endowed with the two independent sets of local continuous symmetries, namely; the usual Yang-Mills (YM) symmetries and non-Yang-Mills (NYM) symmetries. Finally, it is free from the problems connected with the 4D topologically massive models. These features make JP model attractive and worth studying in detail.

The JP model has been explored in many different prospects such as constraint analysis and Hamiltonian formalism [18], establishment of Slavnov-Taylor identities and BRST symmetries [19]. Furthermore, this model is also shown to be ultraviolet finite and renormalizable [20]. We have applied superfield formalism and derived the full set of off-shell nilpotent and absolutely anticommuting BRST as well as anti-BRST symmetry transformations corresponding to the both YM and NYM symmetries of JP model [21, 22]. Within the superfield formalism, we have been able to derive the *proper* (anti-)BRST transformations for the auxiliary field ρ which can neither be deduced by the conventional means of nilpotency and/or absolute anticommutativity of (anti-)BRST symmetries nor generated by the conserved (anti-)BRST charges. At this stage, we would like to point out that the derivation of the proper anti-BRST symmetries have utmost importance because they play a fundamental role in the BRST formalism (see, e.g. [23, 24, 25] for details). In fact, both the symmetries (i.e. BRST and anti-BRST) have been formulated in an independent way [26].

Recently, the (anti-)BRST symmetries for perturbative quantum gravity in curved as well as complex spacetime, in linear as well as in non-linear gauges have been found [27,

28] and a superspace formulation of higher derivative theories [29], Chern-Simons and Yang-Mills theories on deformed superspace [30, 31] within BV formalism have also been established. Moreover, the study of massless and massive fields with totally symmetric arbitrary spin in AdS space has been carried out in the framework of BRST formalism [32].

The main motivations behind our present investigation are as follows. First, the derivation of off-shell nilpotent and absolutely anticommuting (anti-)BRST symmetry transformations corresponding to the combined YM and NYM symmetries of JP model. As, in our recent works (cf. [21, 22]), we have already established the corresponding proper (anti-)BRST symmetry transformations, individually, for both the YM and NYM cases, within the framework of superfield formalism. Second, to establish the Curci-Ferrari (CF) conditions in the case of combined symmetries. These CF conditions are hallmark of any non-Abelian 1-form gauge theories [23] and have a close connection with gerbes [33], within the framework of BRST formalism. Third, to procure appropriate coupled Lagrangian densities which respect the (anti-)BRST symmetries derived from augmented superfield approach. Finally, to point out the role of auxiliary field ρ , which is very special to this model (cf. [18, 21] for details).

This paper is organized in the following manner. In Section 2, we recapitulate the underlying symmetries of 3D JP model. We derive the off-shell nilpotent and absolutely anticommuting (anti-)BRST symmetries corresponding to the combined YM and NYM symmetries of JP model, within the framework of superfield formalism, in Section 3. Section 4 contains the derivation of coupled Lagrangian densities that respect the preceding (anti-)BRST symmetries. The conservation of (anti-)BRST charges is shown in Section 5. We also discuss about the novel observations of our present study in this section. Section 6 is devoted for the discussions of ghost symmetries and BRST algebra. In Section 7, we provide a bird's-eye view on the role of auxiliary field in the context of various massive models. Finally, in Section 8, we make some concluding remarks.

In Appendix A, we show the nilpotency and absolute anticommutativity of the (anti-)BRST charges within the framework of augmented superfield formalism. We also capture (anti-)BRST invariance of coupled Lagrangian densities in the superfield framework.

Conventions and notation: We adopt here the conventions and notation such that the 3D flat Minkowski metric $\eta_{\mu\nu} = \text{diag}(+1, -1, -1)$ and the 3D totally antisymmetric Levi-Civita tensor $\varepsilon_{\mu\nu\eta}$ satisfies $\varepsilon_{\mu\nu\eta} \varepsilon^{\mu\nu\eta} = -3!$, $\varepsilon_{\mu\nu\eta} \varepsilon^{\mu\nu\kappa} = -2! \delta_\eta^\kappa$, etc. with $\varepsilon_{012} = -\varepsilon^{012} = +1$. The Greek indices $\mu, \nu, \eta, \dots = 0, 1, 2$ correspond to the 3D spacetime directions and Latin indices $i, j, k, \dots = 1, 2$ correspond to the space directions only. The dot and cross product between two non-null vectors P and Q in the $SU(N)$ Lie algebraic space are defined as $P \cdot Q = P^a Q^a$, $P \times Q = f^{abc} P^a Q^b T^c$. The $SU(N)$ generators T^a (with $a, b, c, \dots = N^2 - 1$) follow the commutation relation $[T^a, T^b] = i f^{abc} T^c$ where the structure constants f^{abc} are chosen to be totally antisymmetric in a, b, c for the semi-simple $SU(N)$ Lie algebra [34].

2 Preliminaries: Jackiw-Pi model

We start off with the massive, non-Abelian, gauge invariant Jackiw-Pi model in $(2+1)$ -dimensions of spacetime. The Lagrangian density of this model is given by [17, 21]

$$\begin{aligned}\mathcal{L}_0 &= -\frac{1}{4} F_{\mu\nu} \cdot F^{\mu\nu} - \frac{1}{4} (G_{\mu\nu} + g F_{\mu\nu} \times \rho) \cdot (G^{\mu\nu} + g F^{\mu\nu} \times \rho) \\ &+ \frac{m}{2} \varepsilon^{\mu\nu\eta} F_{\mu\nu} \cdot \phi_\eta,\end{aligned}\tag{1}$$

where the 2-form $F^{(2)} = dA^{(1)} + ig(A^{(1)} \wedge A^{(1)}) = \frac{1}{2!} (dx^\mu \wedge dx^\nu) F_{\mu\nu} \cdot T$ defines the curvature tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - g(A_\mu \times A_\nu)$ for the non-Abelian 1-form $[A^{(1)} = dx_\mu A^\mu \cdot T]$ gauge field $A_\mu = A_\mu \cdot T$ where $d = dx^\mu \partial_\mu$ is the exterior derivative (with $d^2 = 0$). Similarly, another 2-form $G^{(2)} = d\phi^{(1)} + ig(A^{(1)} \wedge \phi^{(1)}) + ig(\phi^{(1)} \wedge A^{(1)}) = \frac{1}{2!} (dx^\mu \wedge dx^\nu) G_{\mu\nu} \cdot T$ defines the curvature tensor $G_{\mu\nu} = D_\mu \phi_\nu - D_\nu \phi_\mu$ corresponding* to 1-form $[\phi^{(1)} = dx^\mu \phi_\mu \cdot T]$ vector field $\phi_\mu = \phi_\mu \cdot T$. In the above, the vector fields A_μ and ϕ_μ have opposite parity thus the JP model becomes parity invariant, ρ is an auxiliary field, g is the coupling constant and m defines the mass parameter.

2.1 Local gauge symmetries: YM and NYM

The above Lagrangian density respects two sets of local and continuous gauge symmetry transformations, namely; YM gauge transformations (δ_1) and NYM gauge transformations (δ_2). These symmetry transformations are [21, 22]

$$\begin{aligned}\delta_1 A_\mu &= D_\mu \Lambda, & \delta_1 \phi_\mu &= -g(\phi_\mu \times \Lambda), & \delta_1 \rho &= -g(\rho \times \Lambda), \\ \delta_1 F_{\mu\nu} &= -g(F_{\mu\nu} \times \Lambda), & \delta_1 G_{\mu\nu} &= -g(G_{\mu\nu} \times \Lambda),\end{aligned}\tag{2}$$

$$\delta_2 A_\mu = 0, \quad \delta_2 \phi_\mu = D_\mu \Sigma, \quad \delta_2 \rho = +\Sigma, \quad \delta_2 F_{\mu\nu} = 0, \quad \delta_2 G_{\mu\nu} = -g(F_{\mu\nu} \times \Sigma),\tag{3}$$

where $\Lambda \equiv \Lambda \cdot T$ and $\Sigma \equiv \Sigma \cdot T$ are the $SU(N)$ valued local gauge parameters corresponding to the YM and NYM gauge transformations, respectively. Under the above local and infinitesimal gauge transformations the Lagrangian density (1) transforms as

$$\delta_1 \mathcal{L}_0 = 0, \quad \delta_2 \mathcal{L}_0 = \partial_\mu \left[\frac{m}{2} \varepsilon^{\mu\nu\eta} F_{\nu\eta} \cdot \Sigma \right].\tag{4}$$

As a consequence, the action integral $S = \int d^3x \mathcal{L}_0$ remains invariant under both the gauge transformations (δ_1 and δ_2) for the physically well-defined fields which vanish off rapidly at infinity. We would like to point out that in order to maintain the NYM symmetry, we have to have the auxiliary field ρ in the theory (cf. Section 7 for details).

*The covariant derivative is defined as $D_\mu * = \partial_\mu * - g(A_\mu \times *)$.

2.2 Combined gauge symmetry

In the above, we have seen that both the YM and NYM transformations are the symmetries of the theory. Thus, the combination of the above symmetries [i.e. $(\delta = \delta_1 + \delta_2)$] would also be the symmetry of theory. Under the combined gauge transformation δ , namely;

$$\begin{aligned}\delta A_\mu &= D_\mu \Lambda, & \delta \phi_\mu &= D_\mu \Sigma - g(\phi_\mu \times \Lambda), & \delta \rho &= \Sigma - g(\rho \times \Lambda), \\ \delta F_{\mu\nu} &= -g(F_{\mu\nu} \times \Lambda), & \delta G_{\mu\nu} &= -g(G_{\mu\nu} \times \Lambda) - g(F_{\mu\nu} \times \Sigma),\end{aligned}\quad (5)$$

the Lagrangian density (1) remains quasi-invariant. To be more specific, the Lagrangian density transforms to a total spacetime derivative

$$\delta \mathcal{L}_0 = \partial_\mu \left[\frac{m}{2} \varepsilon^{\mu\nu\eta} F_{\nu\eta} \cdot \Sigma \right]. \quad (6)$$

Thus, the action integral remains invariant (i.e. $\delta S = \delta \int d^3x \mathcal{L}_0 = 0$) under the combined symmetry (δ) , too.

3 (Augmented) superfield approach

We apply Bonora-Tonin's (BT) superfield approach to the BRST formalism [35, 36], to derive the off-shell nilpotent and absolutely anticommuting (anti-) BRST symmetry transformations for the 1-form gauge field A_μ and corresponding (anti-)ghost fields $(\bar{C})C$.

3.1 (Anti-)BRST symmetries: Gauge and (anti-)ghost fields

For this purpose, we generalize 1-form connection $A^{(1)}$ (and corresponding 2-form curvature $F^{(2)}$) and exterior derivative d onto the $(3, 2)$ -dimensional supermanifold, as

$$\begin{aligned}d \rightarrow \tilde{d} &= dZ^M \partial_M = dx^\mu \partial_\mu + d\theta \partial_\theta + d\bar{\theta} \partial_{\bar{\theta}}, & \tilde{d}^2 &= 0, \\ A^{(1)} \rightarrow \tilde{\mathcal{A}}^{(1)} &= dZ^M \tilde{\mathcal{A}}_M = dx^\mu \tilde{\mathcal{A}}_\mu(x, \theta, \bar{\theta}) + d\theta \tilde{\tilde{\mathcal{F}}}(x, \theta, \bar{\theta}) + d\bar{\theta} \tilde{\mathcal{F}}(x, \theta, \bar{\theta}), \\ F^{(2)} \rightarrow \tilde{\mathcal{F}}^{(2)} &= \frac{1}{2!} (dx^M \wedge dx^N) \tilde{\mathcal{F}}_{MN} = \tilde{d}\tilde{\mathcal{A}}^{(1)} + ig(\tilde{\mathcal{A}}^{(1)} \wedge \tilde{\mathcal{A}}^{(1)}),\end{aligned}\quad (7)$$

where $Z^M = (x^\mu, \theta, \bar{\theta})$ are superspace coordinates characterizing the $(3, 2)$ -dimensional supermanifold. In the above expression, θ and $\bar{\theta}$ are the Grassmannian variables (with $\theta^2 = \bar{\theta}^2 = 0, \theta\bar{\theta} + \bar{\theta}\theta = 0$) and $\partial_\theta, \partial_{\bar{\theta}}$ are corresponding Grassmannian derivatives. We also generalize 3D gauge field $[A_\mu(x)]$ and (anti-)ghost fields $[(\bar{C})C(x)]$ of the theory to their corresponding superfields onto the $(3, 2)$ -dimensional supermanifold.

Now, these superfields can be expanded along the Grassmannian directions, in terms of the basic fields (A_μ, C, \bar{C}) and secondary fields $(R_\mu, \bar{R}_\mu, S_\mu, B_1, B_2, \bar{B}_1, \bar{B}_2, s, \bar{s})$, in the following manner,

$$\begin{aligned}\tilde{\mathcal{A}}_\mu(x, \theta, \bar{\theta}) &= A_\mu(x) + \theta \bar{R}_\mu(x) + \bar{\theta} R_\mu(x) + i\theta\bar{\theta} S_\mu(x), \\ \tilde{\mathcal{F}}(x, \theta, \bar{\theta}) &= C(x) + i\theta \bar{B}_1(x) + i\bar{\theta} B_1(x) + i\theta\bar{\theta} s(x), \\ \tilde{\tilde{\mathcal{F}}}(x, \theta, \bar{\theta}) &= \bar{C}(x) + i\theta \bar{B}_2(x) + i\bar{\theta} B_2(x) + i\theta\bar{\theta} \bar{s}(x).\end{aligned}\quad (8)$$

Here $\tilde{\mathcal{A}}_\mu(x, \theta, \bar{\theta})$, $\tilde{\mathcal{F}}(x, \theta, \bar{\theta})$, $\tilde{\bar{\mathcal{F}}}(x, \theta, \bar{\theta})$ are superfields corresponding to the basic fields $A_\mu(x)$, $C(x)$ and $\bar{C}(x)$, respectively. Now these secondary fields, in the above expression, can be determined in terms of the basic and auxiliary fields of the underlying theory through the application of horizontality condition (HC) (cf. [35, 36] for details). This HC can be mathematically expressed in the following fashion

$$d A^{(1)} + i g (A^{(1)} \wedge A^{(1)}) = \tilde{d} \tilde{\mathcal{A}}^{(1)} + i g (\tilde{\mathcal{A}}^{(1)} \wedge \tilde{\mathcal{A}}^{(1)}) \iff F^{(2)} = \tilde{\mathcal{F}}^{(2)}. \quad (9)$$

Exploiting the above HC, we obtain the following relationships among the basic, auxiliary and secondary fields of the theory

$$\begin{aligned} R_\mu &= D_\mu C, \quad \bar{R}_\mu = D_\mu \bar{C}, \quad B_1 = -\frac{i}{2} g (C \times C), \quad \bar{B}_2 = -\frac{i}{2} g (\bar{C} \times \bar{C}), \\ B + \bar{B} &= -i g (C \times \bar{C}), \quad s = -g (\bar{B} \times C), \quad \bar{s} = +g (B \times \bar{C}), \\ S_\mu &= D_\mu B + i g (D_\mu C \times \bar{C}) \equiv -D_\mu \bar{B} - i g (D_\mu \bar{C} \times C), \end{aligned} \quad (10)$$

where we have chosen $\bar{B}_1 = \bar{B}$ and $B_2 = B$.

Substituting the relationships (10) into the super-expansion of superfields in (8), we procure following explicit expansions

$$\begin{aligned} \tilde{\mathcal{A}}_\mu^{(h)}(x, \theta, \bar{\theta}) &= A_\mu(x) + \theta D_\mu \bar{C}(x) + \bar{\theta} D_\mu C(x) + \theta \bar{\theta} [i D_\mu B - g (D_\mu C \times \bar{C})](x) \\ &\equiv A_\mu(x) + \theta (s_{ab} A_\mu(x)) + \bar{\theta} (s_b A_\mu(x)) + \theta \bar{\theta} (s_b s_{ab} A_\mu(x)), \\ \tilde{\mathcal{F}}^{(h)}(x, \theta, \bar{\theta}) &= C(x) + \theta (i \bar{B}(x)) + \bar{\theta} \left[\frac{g}{2} (C \times C) \right](x) + \theta \bar{\theta} [-i g (\bar{B} \times C)](x) \\ &\equiv C(x) + \theta (s_{ab} C(x)) + \bar{\theta} (s_b C(x)) + \theta \bar{\theta} (s_b s_{ab} C(x)), \\ \tilde{\bar{\mathcal{F}}}^{(h)}(x, \theta, \bar{\theta}) &= \bar{C}(x) + \theta \left[\frac{g}{2} (\bar{C} \times \bar{C}) \right](x) + \bar{\theta} (i B(x)) + \theta \bar{\theta} [i g (B \times \bar{C})](x) \\ &\equiv \bar{C}(x) + \theta (s_{ab} \bar{C}(x)) + \bar{\theta} (s_b \bar{C}(x)) + \theta \bar{\theta} (s_b s_{ab} \bar{C}(x)). \end{aligned} \quad (11)$$

In the above, the superscript (h) on the superfields denotes the super-expansion of the superfields obtained after the application of HC (9). Thus, from the above expressions, we can easily identify the (anti-)BRST symmetry transformations corresponding to the gauge field A_μ and (anti-)ghost fields $(\bar{C})C$. These transformations are explicitly listed below

$$\begin{aligned} s_b A_\mu &= D_\mu C, \quad s_b C = \frac{g}{2} (C \times C), \quad s_b \bar{B} = -g (\bar{B} \times C), \\ s_b \bar{C} &= i B, \quad s_b B = 0, \end{aligned} \quad (12)$$

$$\begin{aligned} s_{ab} A_\mu &= D_\mu \bar{C}, \quad s_{ab} \bar{C} = \frac{g}{2} (\bar{C} \times \bar{C}), \quad s_{ab} B = -g (B \times \bar{C}), \\ s_{ab} C &= i \bar{B}, \quad s_{ab} \bar{B} = 0. \end{aligned} \quad (13)$$

We point out that, the (anti-)BRST symmetry transformations for the Nakanishi-Lautrup auxiliary fields B and \bar{B} have been derived with the help of absolute anticommutativity and nilpotency properties of the above (anti-)BRST symmetries.

3.2 (Anti-)BRST symmetries for ϕ_μ , β and $\bar{\beta}$

In the previous subsection, we applied BT superfield approach to derive the off-shell nilpotent and absolutely anti-commuting (anti-)BRST symmetry transformations for the gauge field (A_μ) and corresponding (anti-)ghost fields (\bar{C}) C . Now, in order to derive the proper (anti-)BRST symmetries for the vector field (ϕ_μ), corresponding (anti-)ghost fields $[(\bar{\beta})\beta]$ and auxiliary field (ρ), we have to go beyond the BT approach. For this purpose, we have exploited the power and strength of augmented superfield approach.

To derive the (anti-)BRST symmetries for the vector field (ϕ_μ) and corresponding (anti-)ghost fields $[(\bar{\beta})\beta]$, we invoke the following HC

$$\tilde{\mathcal{G}}^{(2)} + \tilde{\mathcal{F}}^{(2)} \equiv G^{(2)} + \mathcal{F}^{(2)}, \quad (14)$$

where $G^{(2)}$, $\mathcal{F}^{(2)}$ are define in the following fashion

$$\begin{aligned} G^{(2)} &= d\phi^{(1)} + ig(A^{(1)} \wedge \phi^{(1)}) + ig(\phi^{(1)} \wedge A^{(1)}) = \frac{1}{2!} (dx^\mu \wedge dx^\nu) G_{\mu\nu}, \\ \mathcal{F}^{(2)} &= -ig(F^{(2)} \wedge \rho^{(0)}) + ig(\rho^{(0)} \wedge F^{(2)}) = \frac{g}{2!} (dx^\mu \wedge dx^\nu) (F_{\mu\nu} \times \rho), \end{aligned} \quad (15)$$

and $\tilde{\mathcal{G}}^{(2)}$, $\tilde{\mathcal{F}}^{(2)}$ are the generalizations of $G^{(2)}$, $\mathcal{F}^{(2)}$ onto the superspace, respectively, which can be explicitly represented in the following manner

$$\begin{aligned} \tilde{\mathcal{G}}^{(2)} &= d\tilde{\Phi}^{(1)} + ig(\tilde{\mathcal{A}}_{(h)}^{(1)} \wedge \tilde{\Phi}^{(1)}) + ig(\tilde{\Phi}^{(1)} \wedge \tilde{\mathcal{A}}_{(h)}^{(1)}), \\ \tilde{\mathcal{F}}^{(2)} &= -ig(\tilde{\mathcal{F}}_{(h)}^{(2)} \wedge \tilde{\rho}^{(0)}) + ig(\tilde{\rho}^{(0)} \wedge \tilde{\mathcal{F}}_{(h)}^{(2)}). \end{aligned} \quad (16)$$

In the above expression, the quantities $\tilde{\mathcal{A}}_{(h)}^{(1)}$, $\tilde{\Phi}^{(1)}$ and $\tilde{\rho}^{(0)}$ are given as

$$\begin{aligned} \tilde{\mathcal{A}}_{(h)}^{(1)}(x, \theta, \bar{\theta}) &= dx^\mu \tilde{\mathcal{A}}_\mu^{(h)}(x, \theta, \bar{\theta}) + d\theta \tilde{\mathcal{F}}^{(h)}(x, \theta, \bar{\theta}) + d\bar{\theta} \tilde{\mathcal{F}}^{(h)}(x, \theta, \bar{\theta}), \\ \tilde{\Phi}^{(1)}(x, \theta, \bar{\theta}) &= dx^\mu \tilde{\Phi}_\mu(x, \theta, \bar{\theta}) + d\theta \tilde{\beta}(x, \theta, \bar{\theta}) + d\bar{\theta} \tilde{\beta}(x, \theta, \bar{\theta}), \\ \tilde{\rho}^{(0)}(x, \theta, \bar{\theta}) &= \tilde{\rho}(x, \theta, \bar{\theta}), \end{aligned} \quad (17)$$

where the sub/super script (h) denotes the quantities obtained after the application of HC. The superfields in the above expression, corresponding to the basic fields $\phi_\mu, \beta, \bar{\beta}$ and ρ of the theory, can be expanded in terms of the secondary fields, as follows

$$\begin{aligned} \tilde{\Phi}_\mu(x, \theta, \bar{\theta}) &= \phi_\mu(x) + \theta \bar{P}_\mu(x) + \bar{\theta} P_\mu(x) + i\theta \bar{\theta} Q_\mu(x), \\ \tilde{\beta}(x, \theta, \bar{\theta}) &= \beta(x) + i\theta \bar{R}_1(x) + i\bar{\theta} R_1(x) + i\theta \bar{\theta} s_1(x), \\ \tilde{\bar{\beta}}(x, \theta, \bar{\theta}) &= \bar{\beta}(x) + i\theta \bar{R}_2(x) + i\bar{\theta} R_2(x) + i\theta \bar{\theta} s_2(x), \\ \tilde{\rho}(x, \theta, \bar{\theta}) &= \rho(x) + \theta \bar{b}(x) + \bar{\theta} b(x) + i\theta \bar{\theta} q(x), \end{aligned} \quad (18)$$

where $P_\mu, \bar{P}_\mu, b, \bar{b}, s_1, s_2$ are fermionic secondary fields and $R_1, \bar{R}_1, R_2, \bar{R}_2, Q_\mu, q$ are bosonic in nature.

Exploiting the above HC (14) which demands that the coefficients of wedge products $(dx^\mu \wedge d\theta)$, $(dx^\mu \wedge d\bar{\theta})$, $(d\theta \wedge d\theta)$, $(d\bar{\theta} \wedge d\bar{\theta})$, $(d\theta \wedge d\bar{\theta})$ set equal to zero. We get following

expressions:

$$\begin{aligned}
\tilde{D}_\mu \tilde{\tilde{\beta}} - \partial_\theta \tilde{\Phi}_\mu - g \left(\tilde{\Phi}_\mu \times \tilde{\tilde{\mathcal{F}}}^{(h)} \right) &= 0, & \partial_\theta \tilde{\tilde{\beta}} - g \left(\tilde{\tilde{\mathcal{F}}}^{(h)} \times \tilde{\tilde{\beta}} \right) &= 0, \\
\tilde{D}_\mu \tilde{\beta} - \partial_{\bar{\theta}} \tilde{\Phi}_\mu - g \left(\tilde{\Phi}_\mu \times \tilde{\mathcal{F}}^{(h)} \right) &= 0, & \partial_{\bar{\theta}} \tilde{\beta} - g \left(\tilde{\mathcal{F}}^{(h)} \times \tilde{\beta} \right) &= 0, \\
\partial_\theta \tilde{\beta} + \partial_{\bar{\theta}} \tilde{\tilde{\beta}} - g \left(\tilde{\tilde{\mathcal{F}}}^{(h)} \times \tilde{\beta} \right) - g \left(\tilde{\mathcal{F}}^{(h)} \times \tilde{\tilde{\beta}} \right) &= 0,
\end{aligned} \tag{19}$$

where $\tilde{D}_\mu \bullet = \partial_\mu \bullet - g(\tilde{\mathcal{A}}_\mu^{(h)} \times \bullet)$. Using the expansion (18) in (19), we get following relationships amongst the basic and secondary fields of the theory, namely;

$$\begin{aligned}
R_1 &= -i g(C \times \beta), & \bar{R}_2 &= -i g(\bar{C} \times \bar{\beta}), & s_1 &= -g(\bar{B} \times \beta) + g(C \times \bar{R}), \\
s_2 &= g(B \times \bar{\beta}) - g(\bar{C} \times R), & R + \bar{R} + i g(C \times \bar{\beta}) + i g(\bar{C} \times \beta) &= 0, \\
P_\mu &= D_\mu \beta - g(\phi_\mu \times C), & D_\mu \bar{R}_2 + i g(D_\mu \bar{C} \times \bar{\beta}) + i g(D_\mu \bar{\beta} \times \bar{C}) &= 0, \\
\bar{P}_\mu &= D_\mu \bar{\beta} - g(\phi_\mu \times \bar{C}), & D_\mu R_1 + i g(D_\mu C \times \beta) + i g(D_\mu \beta \times C) &= 0, \\
Q_\mu &= D_\mu R + g(B \times \phi_\mu) + i g(D_\mu C \times \bar{\beta}) + i g[D_\mu \beta \times \bar{C} - g(\phi_\mu \times C) \times \bar{C}] \\
&\equiv -D_\mu \bar{R} - g(\bar{B} \times \phi_\mu) - i g(D_\mu \bar{C} \times \beta) - i g[D_\mu \bar{\beta} \times C - g(\phi_\mu \times \bar{C}) \times C],
\end{aligned} \tag{20}$$

where we have chosen $\bar{R}_1 = \bar{R}$, $R_2 = R$. Substituting, these values of secondary fields in (18), we have following form of superfield expansions

$$\begin{aligned}
\tilde{\Phi}_\mu^{(h)}(x, \theta, \bar{\theta}) &= \phi_\mu(x) + \theta [D_\mu \bar{\beta} - g(\phi_\mu \times \bar{C})](x) + \bar{\theta} [D_\mu \beta - g(\phi_\mu \times C)](x) \\
&+ \theta \bar{\theta} [i D_\mu R - i g(\phi_\mu \times B) - g(D_\mu C \times \bar{\beta}) - g(D_\mu \beta \times \bar{C}) \\
&+ g^2(\phi_\mu \times C) \times \bar{C}](x) \\
&\equiv \phi(x) + \theta (s_b \phi(x)) + \bar{\theta} (s_{ab} \phi(x)) + \theta \bar{\theta} (s_b s_{ab} \phi(x)), \\
\tilde{\beta}^{(h)}(x, \theta, \bar{\theta}) &= \beta(x) + \theta [i \bar{R}(x)] + \bar{\theta} [g(C \times \beta)](x) \\
&+ \theta \bar{\theta} [-i g(\bar{B} \times \beta) - i g(\bar{R} \times C)](x) \\
&\equiv \beta(x) + \theta (s_b \beta(x)) + \bar{\theta} (s_{ab} \beta(x)) + \theta \bar{\theta} (s_b s_{ab} \beta(x)), \\
\tilde{\tilde{\beta}}^{(h)}(x, \theta, \bar{\theta}) &= \bar{\beta}(x) + \theta [g(\bar{C} \times \bar{\beta})](x) + \bar{\theta} [i R(x)] \\
&+ \theta \bar{\theta} [i g(B \times \bar{\beta}) + i g(R \times \bar{C})](x) \\
&\equiv \bar{\beta}(x) + \theta (s_b \bar{\beta}(x)) + \bar{\theta} (s_{ab} \bar{\beta}(x)) + \theta \bar{\theta} (s_b s_{ab} \bar{\beta}(x)),
\end{aligned} \tag{21}$$

here (h) on the superscript of superfields represents the respective quantities obtained after the application of HC (14). Therefore, (anti-)BRST symmetry transformations for vector field (ϕ_μ) and (anti-)ghost fields $[(\bar{\beta})\beta]$ are obvious from the above super-expansions.

3.3 (Anti-)BRST symmetries for auxiliary field ρ

In order to derive the proper (anti-)BRST symmetry transformations for the auxiliary field ρ , we look for a quantity which remains invariant (or should transform covariantly) under the combined gauge transformations (5). Such gauge invariant quantity will serve a purpose

of ‘physical quantity’ (in some sense) which could be generalized onto the $(3, 2)$ -dimensional supermanifold. Furthermore, being a ‘physical quantity’ it should remain unaffected by the presence of Grassmannian variables when the former is generalized onto the supermanifold. Thus, keeping above in mind, we note that under the combined gauge transformations (5), the quantity $(D_\mu \rho - \phi_\mu)$ transforms covariantly (as the quantities $F_{\mu\nu}$ and $G_{\mu\nu} + g(F_{\mu\nu} \times \rho)$ do). This can be explicitly checked as follows

$$\delta(D_\mu \rho - \phi_\mu) = -g(D_\mu \rho - \phi_\mu) \times \Lambda. \quad (22)$$

Therefore, the above quantity serves our purpose and it can also be expressed in the language of differential forms as follows

$$d\rho^{(0)} + i g (A^{(1)} \wedge \rho^{(0)}) - i g (\rho^{(0)} \wedge A^{(1)}) - \phi^{(1)} = dx^\mu (D_\mu \rho - \phi_\mu), \quad (23)$$

which is clearly a 1-form object. Now, we generalize this 1-form object onto the $(3, 2)$ -dimensional supermanifold and demand that it should remain unaffected by the presence of Grassmannian variables. This, in turn, produces the following HC

$$\begin{aligned} d\rho^{(0)} + i g (A^{(1)} \wedge \rho^{(0)}) - i g (\rho^{(0)} \wedge A^{(1)}) - \phi^{(1)} &\equiv \tilde{d}\tilde{\rho}^{(0)} + i g (\tilde{\mathcal{A}}_{(h)}^{(1)} \wedge \tilde{\rho}^{(0)}) \\ &- i g (\tilde{\rho}^{(0)} \wedge \tilde{\mathcal{A}}_{(h)}^{(1)}) - \tilde{\Phi}_{(h)}^{(1)}. \end{aligned} \quad (24)$$

This HC can also be derived from the integrability of (14) (see e.g., [37] for details on the topic). The r.h.s. of the above HC can be simplified as

$$\begin{aligned} \tilde{d}\tilde{\rho}^{(0)} + i g (\tilde{\mathcal{A}}_{(h)}^{(1)} \wedge \tilde{\rho}^{(0)}) - i g (\tilde{\rho}^{(0)} \wedge \tilde{\mathcal{A}}_{(h)}^{(1)}) - \tilde{\Phi}_{(h)}^{(1)} = \\ dx^\mu \left[\tilde{\mathcal{D}}_\mu \tilde{\rho} - \tilde{\Phi}_\mu^{(h)} \right] + d\theta \left[\partial_\theta \tilde{\rho} - \tilde{\beta}^{(h)} - g \left(\tilde{\tilde{\mathcal{F}}}^{(h)} \times \tilde{\rho} \right) \right] + d\bar{\theta} \left[\partial_{\bar{\theta}} \tilde{\rho} - \tilde{\beta}^{(h)} - g \left(\tilde{\mathcal{F}}^{(h)} \times \tilde{\rho} \right) \right]. \end{aligned} \quad (25)$$

Exploiting (24), and set the coefficients of $d\theta, d\bar{\theta}$ equal to zero, we have the following relationships, namely;

$$\partial_\theta \tilde{\rho} - \tilde{\beta}^{(h)} - g \left(\tilde{\tilde{\mathcal{F}}}^{(h)} \times \tilde{\rho} \right) = 0, \quad \partial_{\bar{\theta}} \tilde{\rho} - \tilde{\beta}^{(h)} - g \left(\tilde{\mathcal{F}}^{(h)} \times \tilde{\rho} \right) = 0. \quad (26)$$

Plugging the values of superfield expansions from (11), (18) and (21) into the above expressions, we get the following relationships amongst the basic and secondary fields

$$\begin{aligned} b &= \beta - g(\rho \times C), \quad \bar{b} = \bar{\beta} - g(\rho \times \bar{C}), \\ q &= R + g(B \times \rho) + i g(\bar{C} \times \beta) - i g^2(\rho \times C) \times \bar{C} \\ &\equiv -\bar{R} - i g(C \times \bar{\beta}) - g(\bar{B} \times \rho) + i g^2(\rho \times \bar{C}) \times C. \end{aligned} \quad (27)$$

We point out that, however, there also exist other relationships but they are same as quoted in equation (20).

Finally, substituting these values of secondary fields into (18), we obtain the following superfield expansion for the super-auxiliary field $\tilde{\rho}(x, \theta, \bar{\theta})$

$$\begin{aligned} \tilde{\rho}^{(h)}(x, \theta, \bar{\theta}) &= \rho(x) + \theta [\bar{\beta} - g(\rho \times \bar{C})](x) + \bar{\theta} [\beta - g(\rho \times C)](x) \\ &+ i\theta \bar{\theta} [R + g(B \times \rho) + i g(\bar{C} \times \beta) - i g^2(\rho \times C) \times \bar{C}](x), \\ &\equiv \rho(x) + \theta (s_b \rho(x)) + \bar{\theta} (s_{ab} \rho(x)) + \theta \bar{\theta} (s_b s_{ab} \rho(x)), \end{aligned} \quad (28)$$

where (h) as the superscript on the generic superfield denotes the corresponding superfield expansion obtained after the application of HC (24). The (anti-) BRST symmetry transformations for the auxiliary field ρ can be easily deduced from the above expansion. Thus, we have derived the proper (anti-)BRST symmetry transformations for the vector field (ϕ_μ) , corresponding (anti-)ghost fields $[(\bar{\beta})\beta]$ and auxiliary field (ρ) within the framework of augmented superfield formalism. Moreover, the (anti-)BRST symmetry transformations for the Nakanishi-Lautrup auxiliary fields R and \bar{R} have been derived with the help of anti-commutativity and nilpotency properties of the (anti-)BRST symmetries. These symmetry transformations are listed below

$$\begin{aligned} s_b \phi_\mu &= D_\mu \beta - g(\phi_\mu \times C), & s_b \beta &= g(C \times \beta), & s_b \rho &= \beta - g(\rho \times C), \\ s_b \bar{\beta} &= i R, & s_b R &= 0, & s_b \bar{R} &= -g(\bar{R} \times C) - g(\bar{B} \times \beta), \end{aligned} \quad (29)$$

$$\begin{aligned} s_{ab} \phi_\mu &= D_\mu \bar{\beta} - g(\phi_\mu \times \bar{C}), & s_{ab} \bar{\beta} &= g(\bar{C} \times \bar{\beta}), & s_{ab} \rho &= \bar{\beta} - g(\rho \times \bar{C}), \\ s_{ab} \beta &= i \bar{R}, & s_{ab} \bar{R} &= 0, & s_{ab} R &= -g(R \times \bar{C}) - g(B \times \bar{\beta}). \end{aligned} \quad (30)$$

These (anti-)BRST symmetry transformations as well as the transformations listed in (12) and (13) are off-shell nilpotent ($s_{(a)b}^2 \Psi = 0$) and absolutely anticommuting $[(s_b s_{ab} + s_{ab} s_b) \Psi = 0]$ in nature. Here Ψ represents any generic field of the theory. These properties (i.e. nilpotency and anticommutativity) are two key ingredients of the BRST formalism. The anticommutativity property for the vector fields $(\phi_\mu$ and $A_\mu)$ and auxiliary field (ρ) is satisfied only on the constrained surface parametrized by the CF conditions (cf. (32) below). For instance, one can check that

$$\begin{aligned} \{s_b, s_{ab}\} A_\mu &= i D_\mu [B + \bar{B} + i(C \times \bar{C})], \\ \{s_b, s_{ab}\} \phi_\mu &= i D_\mu [R + \bar{R} + i g(C \times \bar{\beta}) + i g(\bar{C} \times \beta)] + i g[B + \bar{B} + i g(C \times \bar{C})] \times \phi_\mu, \\ \{s_b, s_{ab}\} \rho &= i [R + \bar{R} + i g(C \times \bar{\beta}) + i g(\bar{C} \times \beta)] + i g[B + \bar{B} + i g(C \times \bar{C})] \times \rho, \end{aligned} \quad (31)$$

whereas, for all the *rest* of the fields (of our present 3D JP model), the absolute anticommutativity property (i.e. $\{s_b, s_{ab}\} \Psi = 0$) is valid *without* invoking the CF type conditions.

Before, we wrap up this section, some crucial points are in order. First and foremost, a very careful look at (10) and (20) reveals, respectively, the existence of two sets of Curci-Ferrari (CF) type conditions, namely;

$$\begin{aligned} (i) \quad & B + \bar{B} + i g(C \times \bar{C}) = 0, \\ (ii) \quad & R + \bar{R} + i g(C \times \bar{\beta}) + i g(\bar{C} \times \beta) = 0. \end{aligned} \quad (32)$$

These conditions are key signatures of any p -form gauge theory when the latter is discussed within the framework of BRST formalism. In our case, the above mentioned CF conditions emerge very naturally within the framework of (augmented) superfield formalism. In fact, CF conditions (i) and (ii) emerge from the HC (9) and (14), respectively, when we set the coefficients of $(d\theta \wedge d\bar{\theta})$ equal to zero. Second, the absolute anticommutativity of (anti-)BRST symmetries is ensured by these CF type conditions. Third, these CF type conditions are (anti-)BRST invariant. Finally, these CF type conditions play a crucial role in the derivation of the coupled (but equivalent) Lagrangian densities. We have discussed this aspect, in detail, in our next section.

4 Coupled Lagrangian densities

In this section, we construct the coupled (but equivalent) Lagrangian densities which respect nilpotent as well as anticommuting (anti-)BRST symmetry transformations derived in the previous section (cf. Section 3). In order to proceed further, a few important points are in order. First, the mass dimensions (in natural units $c = \hbar = 1$) of the various fields in our present 3D theory are: $[A_\mu] = [\phi_\mu] = [C] = [\bar{C}] = [\beta] = [\bar{\beta}] = [M]^{\frac{1}{2}}$, $[B] = [\bar{B}] = [R] = [\bar{R}] = [M]^{\frac{3}{2}}$, $[\rho] = [M]^{-\frac{1}{2}}$, and the coupling constant g has the mass dimension $[g] = [M]^{\frac{1}{2}}$. Second, the fermionic (anti-)ghost fields $(\bar{C})C$ and $(\bar{\beta})\beta$ carry ghost numbers (∓ 1) , respectively whereas rest of the (bosonic) fields carry ghost number equal to zero. Third, the nilpotent (anti-)BRST transformations increase the mass dimension by one unit when they operate on any generic field of the theory. In other words, we can say that the (anti-)BRST transformations carry mass dimension equal one (in natural units). Fourth, the (anti-)BRST transformations (decrease)increase the ghost number by one unit when they act on any field of the theory. This means that (anti-)BRST transformations carry ghost number (∓ 1) , respectively. These points are very important in constructing the (anti-)BRST invariant coupled Lagrangian densities.

Exploiting the basic tenets of the BRST formalism, the most appropriate (anti-)BRST invariant Lagrangian densities that can be written in terms of nilpotent and absolutely anticommuting (anti-)BRST symmetry transformations are as follows [37]

$$\begin{aligned}\mathcal{L}_b &= \mathcal{L}_0 + s_b s_{ab} \left[\frac{i}{2} A_\mu \cdot A^\mu + C \cdot \bar{C} + \frac{i}{2} \phi_\mu \cdot \phi^\mu + \frac{1}{2} \beta \cdot \bar{\beta} \right], \\ \mathcal{L}_{\bar{b}} &= \mathcal{L}_0 - s_{ab} s_b \left[\frac{i}{2} A_\mu \cdot A^\mu + C \cdot \bar{C} + \frac{i}{2} \phi_\mu \cdot \phi^\mu + \frac{1}{2} \beta \cdot \bar{\beta} \right],\end{aligned}\quad (33)$$

where \mathcal{L}_0 is our starting gauge invariant Lagrangian density (1). We would like to emphasize that each term in the square brackets is Lorentz scalar and chosen in such a way that they have ghost number zero and mass dimension one (in natural units). Moreover, the (constant) factors in front of each term are picked for the algebraic convenience. Utilizing the off-shell nilpotent (anti-) BRST transformations from (12), (13), (29) and (30), we obtain the following explicit Lagrangian densities, namely;

$$\begin{aligned}\mathcal{L}_b &= -\frac{1}{4} F_{\mu\nu} \cdot F^{\mu\nu} - \frac{1}{4} (G_{\mu\nu} + g F_{\mu\nu} \times \rho) \cdot (G^{\mu\nu} + g F^{\mu\nu} \times \rho) + \frac{m}{2} \varepsilon^{\mu\nu\eta} F_{\mu\nu} \cdot \phi_\eta \\ &+ \frac{1}{2} [B \cdot B + \bar{B} \cdot \bar{B}] + B \cdot (\partial^\mu A_\mu) + \frac{1}{2} [R + ig(C \times \bar{\beta})] \cdot [R + ig(C \times \bar{\beta})] \\ &+ [R + ig(C \times \bar{\beta})] \cdot (D^\mu \phi_\mu) - i \partial_\mu \bar{C} \cdot D^\mu C - i D_\mu \bar{\beta} \cdot D^\mu \beta, \\ \mathcal{L}_{\bar{b}} &= -\frac{1}{4} F_{\mu\nu} \cdot F^{\mu\nu} - \frac{1}{4} (G_{\mu\nu} + g F_{\mu\nu} \times \rho) \cdot (G^{\mu\nu} + g F^{\mu\nu} \times \rho) + \frac{m}{2} \varepsilon^{\mu\nu\eta} F_{\mu\nu} \cdot \phi_\eta \\ &+ \frac{1}{2} [B \cdot B + \bar{B} \cdot \bar{B}] - \bar{B} \cdot (\partial^\mu A_\mu) + \frac{1}{2} [\bar{R} + ig(\bar{C} \times \beta)] \cdot [\bar{R} + ig(\bar{C} \times \beta)] \\ &- [\bar{R} + ig(\bar{C} \times \beta)] \cdot (D^\mu \phi_\mu) - i D_\mu \bar{C} \cdot \partial^\mu C - i D_\mu \bar{\beta} \cdot D^\mu \beta,\end{aligned}\quad (34)$$

where B, \bar{B} and R, \bar{R} are the Nakanishi-Lautrup type auxiliary fields. These Lagrangian

densities are coupled because these Nakanishi-Lautrup auxiliary fields B, \bar{B} and R, \bar{R} are related through the CF conditions (32).

It can be checked that the (anti-)BRST transformations $(s_{(a)b})$ leave the above Lagrangian densities quasi-invariant. To be more specific, under the operations of nilpotent (anti-)BRST transformations, the Lagrangian densities $(\mathcal{L}_{\bar{b}})\mathcal{L}_b$ transform to a total space-time derivative, in the following fashion, respectively

$$\begin{aligned} s_{ab}\mathcal{L}_{\bar{b}} &= \partial_\mu \left[\frac{m}{2} \varepsilon^{\mu\nu\eta} F_{\nu\eta} \cdot \bar{\beta} - \bar{B} \cdot (D^\mu \bar{C}) - \bar{R} \cdot D^\mu \bar{\beta} - ig(\bar{C} \times \beta) \cdot D^\mu \bar{\beta} \right], \\ s_b\mathcal{L}_b &= \partial_\mu \left[\frac{m}{2} \varepsilon^{\mu\nu\eta} F_{\nu\eta} \cdot \beta + B \cdot (D^\mu C) + R \cdot D^\mu \beta + ig(C \times \bar{\beta}) \cdot D^\mu \beta \right]. \end{aligned} \quad (35)$$

Thus, the action integral corresponding to the above Lagrangian densities remain invariant under $(s_{(a)b})$. Furthermore, it is interesting to note that the following variations are true:

$$\begin{aligned} s_{ab}\mathcal{L}_b &= \partial_\mu \left[\frac{m}{2} \varepsilon^{\mu\nu\eta} F_{\nu\eta} \cdot \bar{\beta} + B \cdot \partial^\mu \bar{C} + (R + igC \times \bar{\beta}) \cdot D^\mu \bar{\beta} \right] \\ &\quad - \left[D_\mu (B + \bar{B} + igC \times \bar{C}) \right] \cdot \partial^\mu \bar{C} - \left[D_\mu (R + \bar{R} + igC \times \bar{\beta} + ig\bar{C} \times \beta) \right] \cdot D^\mu \bar{\beta} \\ &\quad - g \left[R + ig(C \times \bar{\beta}) + D^\mu \phi_\mu \right] \cdot \left[(B + \bar{B} + igC \times \bar{C}) \times \bar{\beta} \right], \\ s_b\mathcal{L}_{\bar{b}} &= \partial_\mu \left[\frac{m}{2} \varepsilon^{\mu\nu\eta} F_{\nu\eta} \cdot \beta - \bar{B} \cdot \partial^\mu C - (\bar{R} + ig\bar{C} \times \beta) \cdot D^\mu \beta \right] \\ &\quad + \left[D_\mu (B + \bar{B} + igC \times \bar{C}) \right] \cdot \partial^\mu C + \left[D_\mu (R + \bar{R} + igC \times \bar{\beta} + ig\bar{C} \times \beta) \right] \cdot D^\mu \beta \\ &\quad - g \left[\bar{R} + ig(\bar{C} \times \beta) - D^\mu \phi_\mu \right] \cdot \left[(B + \bar{B} + igC \times \bar{C}) \times \beta \right]. \end{aligned} \quad (36)$$

Therefore, it is evident from the above variations that the Lagrangian densities \mathcal{L}_b and $\mathcal{L}_{\bar{b}}$ also respect the anti-BRST (s_{ab}) and BRST (s_b) transformations, respectively only on the constrained hypersurface defined by the CF conditions (32). As a result, both the Lagrangian densities are equivalent and they respect BRST as well as anti-BRST symmetries on the constrained hypersurface spanned by CF conditions [cf. (32)].

5 Conserved charges: Novel observations

In our previous section, we have seen that the coupled Lagrangian densities (and corresponding actions) respect the off-shell nilpotent and continuous (anti-)BRST symmetry transformations. As a consequence, according to Noether's theorem, the invariance of the actions under the continuous (anti-) BRST transformations lead to the following conserved

(anti-)BRST currents ($J_{(a)b}^\mu$), namely;

$$\begin{aligned}
J_{ab}^\mu &= -(D_\nu \bar{C}) \cdot \left[F^{\mu\nu} - g(G^{\mu\nu} + gF^{\mu\nu} \times \rho) \times \rho - m \varepsilon^{\mu\nu\eta} \phi_\eta \right] - \bar{B} \cdot (D^\mu \bar{C}) \\
&\quad - \frac{i}{2} g(\bar{C} \times \bar{C}) \cdot \partial^\mu C - (D_\nu \bar{\beta}) \cdot \left(G^{\mu\nu} + gF^{\mu\nu} \times \rho \right) + g(\phi_\nu \times \bar{C}) \cdot \left(G^{\mu\nu} + gF^{\mu\nu} \times \rho \right) \\
&\quad + g(\phi^\mu \times \bar{C}) \cdot (\bar{R} + ig\bar{C} \times \beta) - \bar{R} \cdot D^\mu \bar{\beta} - ig(\bar{C} \times \bar{\beta}) \cdot D^\mu \beta - \frac{m}{2} \varepsilon^{\mu\nu\eta} F_{\nu\eta} \cdot \bar{\beta}, \\
J_b^\mu &= -(D_\nu C) \cdot \left[F^{\mu\nu} - g(G^{\mu\nu} + gF^{\mu\nu} \times \rho) \times \rho - m \varepsilon^{\mu\nu\eta} \phi_\eta \right] + B \cdot (D^\mu C) \\
&\quad + \frac{i}{2} g(C \times C) \cdot \partial^\mu \bar{C} - (D_\nu \beta) \cdot \left(G^{\mu\nu} + gF^{\mu\nu} \times \rho \right) + g(\phi_\nu \times C) \cdot \left(G^{\mu\nu} + gF^{\mu\nu} \times \rho \right) \\
&\quad - g(\phi^\mu \times C) \cdot (R + igC \times \bar{\beta}) + R \cdot D^\mu \beta + ig(C \times \beta) \cdot D^\mu \bar{\beta} - \frac{m}{2} \varepsilon^{\mu\nu\eta} F_{\nu\eta} \cdot \beta. \quad (37)
\end{aligned}$$

One can check that the conservation (i.e. $\partial_\mu J_b^\mu = 0$) of BRST current (J_b^μ) can be proven by exploiting the Euler-Lagrange (E-L) equations of motion that are derived from the Lagrangian density \mathcal{L}_b . These E-L equations are as listed below:

$$\begin{aligned}
&D_\mu F^{\mu\nu} - g D_\mu [(G^{\mu\nu} + gF^{\mu\nu} \times \rho) \times \rho] + g(G^{\mu\nu} + gF^{\mu\nu} \times \rho) \times \phi_\mu - m \varepsilon^{\mu\nu\eta} (D_\mu \phi_\eta) \\
&- \partial^\nu B - ig(\partial^\nu \bar{C} \times C) + g(R + igC \times \bar{\beta}) \times \phi^\nu + ig(\bar{\beta} \times D^\nu \beta) - ig(\beta \times D^\nu \bar{\beta}) = 0, \\
&D_\mu [G^{\mu\nu} + g(F^{\mu\nu} \times \rho)] - D^\nu [R + ig(C \times \bar{\beta})] - \frac{m}{2} \varepsilon^{\mu\nu\kappa} F_{\mu\kappa} = 0, \\
&[G^{\mu\nu} + g(F^{\mu\nu} \times \rho)] \times F_{\mu\nu} = 0, \quad R + ig(C \times \bar{\beta}) + D_\mu \phi^\mu = 0, \quad B = -(\partial_\mu A^\mu), \\
&\partial_\mu (D^\mu C) = 0, \quad D_\mu (\partial^\mu \bar{C}) = 0, \quad D_\mu (D^\mu \beta) = 0, \quad D_\mu (D^\mu \bar{\beta}) = 0, \quad (38)
\end{aligned}$$

$$\begin{aligned}
&D_\mu F^{\mu\nu} - g D_\mu [(G^{\mu\nu} + gF^{\mu\nu} \times \rho) \times \rho] + g(G^{\mu\nu} + gF^{\mu\nu} \times \rho) \times \phi_\mu - m \varepsilon^{\mu\nu\eta} (D_\mu \phi_\eta) \\
&+ \partial^\nu \bar{B} + ig(\partial^\nu C \times \bar{C}) - g(\bar{R} + ig\bar{C} \times \beta) \times \phi^\nu + ig(\bar{\beta} \times D^\nu \beta) - ig(\beta \times D^\nu \bar{\beta}) = 0, \\
&D_\mu [G^{\mu\nu} + g(F^{\mu\nu} \times \rho)] + D^\nu [\bar{R} + ig(\bar{C} \times \beta)] - \frac{m}{2} \varepsilon^{\mu\nu\kappa} F_{\mu\kappa} = 0, \\
&[G^{\mu\nu} + g(F^{\mu\nu} \times \rho)] \times F_{\mu\nu} = 0, \quad \bar{R} + ig(\bar{C} \times \beta) - D_\mu \phi^\mu = 0, \quad \bar{B} = (\partial_\mu A^\mu), \\
&D_\mu (\partial^\mu C) = 0, \quad \partial_\mu (D^\mu \bar{C}) = 0, \quad D_\mu (D^\mu \beta) = 0, \quad D_\mu (D^\mu \bar{\beta}) = 0, \quad (39)
\end{aligned}$$

which emerge from the Lagrangian density $\mathcal{L}_{\bar{b}}$.

Exploiting the above E-L equations of motion (cf. (38) and (39)), the conserved currents $J_{(a)b}^\mu$ can be written in simpler forms as:

$$\begin{aligned}
J_{ab}^\mu &= -\partial_\nu \left([F^{\mu\nu} - g(G^{\mu\nu} + gF^{\mu\nu} \times \rho) \times \rho - m \varepsilon^{\mu\nu\eta} \phi_\eta] \cdot \bar{C} + [G^{\mu\nu} + gF^{\mu\nu} \times \rho] \cdot \bar{\beta} \right) \\
&\quad + (\partial^\mu \bar{B}) \cdot \bar{C} - \bar{B} \cdot (D^\mu \bar{C}) + \frac{i}{2} g(\bar{C} \times \bar{C}) \cdot \partial^\mu C - (\bar{R} + ig\bar{C} \times \beta) \cdot (D^\mu \bar{\beta}) \\
&\quad + D^\mu (\bar{R} + ig\bar{C} \times \beta) \cdot \bar{\beta}, \\
J_b^\mu &= -\partial_\nu \left([F^{\mu\nu} - g(G^{\mu\nu} + gF^{\mu\nu} \times \rho) \times \rho - m \varepsilon^{\mu\nu\eta} \phi_\eta] \cdot C + [G^{\mu\nu} + gF^{\mu\nu} \times \rho] \cdot \beta \right) \\
&\quad + B \cdot (D^\mu C) - (\partial^\mu B) \cdot C - \frac{i}{2} g(C \times C) \cdot \partial^\mu \bar{C} + (R + igC \times \bar{\beta}) \cdot (D^\mu \beta) \\
&\quad - D^\mu (R + igC \times \bar{\beta}) \cdot \beta. \quad (40)
\end{aligned}$$

Now, the proof of conservation laws ($\partial_\mu J_{(a)b}^\mu = 0$) is quite straightforward. The temporal components (i.e. $\int d^2x J_{(a)b}^0 = Q_{(a)b}$) of the above conserved currents ($J_{(a)b}^\mu$) lead to the following conserved (i.e. $\dot{Q}_{(a)b} = 0$) (anti-)BRST charges ($Q_{(a)b}$), namely;

$$\begin{aligned} Q_{ab} &= - \int d^2x \left[\bar{B} \cdot (D^0 \bar{C}) - (\partial^0 \bar{B}) \cdot \bar{C} - \frac{i}{2} g (\bar{C} \times \bar{C}) \cdot \partial^0 C + (\bar{R} + ig \bar{C} \times \beta) \cdot (D^0 \bar{\beta}) \right. \\ &\quad \left. - D^0 (\bar{R} + ig \bar{C} \times \beta) \cdot \bar{\beta} \right], \\ Q_b &= \int d^2x \left[B \cdot (D^0 C) - (\partial^0 B) \cdot C - \frac{i}{2} g (C \times C) \cdot \partial^0 \bar{C} + (R + ig C \times \bar{\beta}) \cdot (D^0 \beta) \right. \\ &\quad \left. - D^0 (R + ig C \times \bar{\beta}) \cdot \beta \right]. \end{aligned} \quad (41)$$

It turns out that the conserved, nilpotent ($Q_{(a)b}^2 = 0$, see below) and anticommuting ($Q_b Q_{ab} + Q_{ab} Q_b = 0$, see below) (anti-)BRST charges are the generators of the (anti-)BRST symmetry transformations, respectively. For the sake of brevity, these transformations can be obtained by exploiting the following symmetry properties:

$$s_b \Psi = -i [\Psi, Q_b]_{\pm}, \quad s_{ab} \Psi = -i [\Psi, Q_{ab}]_{\pm}, \quad \Psi = A_\mu, \phi_\mu, C, \bar{C}, \beta, \bar{\beta} \quad (42)$$

The (\pm) signs as the subscript on the square brackets represent (anti)commutators corresponding to the generic field Ψ being (fermionic)bosonic in nature (see, e.g. [38] for details). The (anti-)BRST transformations of the Nakanishi-Lautrup auxiliary fields B, \bar{B}, R, \bar{R} have been derived from the basic requirements (i.e. nilpotency and/or absolute anticommutativity properties) of the (anti-)BRST symmetry transformations.

It is worthwhile to mention that, even though, the (anti-)BRST charges ($Q_{(a)b}$) are conserved, nilpotent as well as anticommuting in nature (see below), they are unable to generate the proper (anti-)BRST transformations (i.e. $s_b \rho = \beta - g(\rho \times C)$ and $s_{ab} \rho = \bar{\beta} - g(\rho \times \bar{C})$) of the auxiliary field ρ . Furthermore, the nilpotency and absolute anticommutativity properties of the (anti-)BRST transformations also fail to produce the transformations of ρ . This is one of the novel observations of our present endeavor. Although, we have derived these transformations by exploiting the power and strength of the augmented superfield formalism which produces the off-shell nilpotent ($s_{(a)b}^2 = 0$) as well as absolutely anticommuting ($s_b s_{ab} + s_{ab} s_b = 0$) (anti-)BRST symmetry transformations for *all* the basic and auxiliary fields of the theory.

The nilpotency ($Q_{(a)b}^2 = 0$) of the (anti-)BRST charges reflects the fermionic nature whereas the anticommutativity ($Q_b Q_{ab} + Q_{ab} Q_b = 0$) shows that the (anti-)BRST charges are linearly independent of each other. These properties can be verified in the following straightforward manner:

$$\begin{aligned} s_b Q_b &= -i \{Q_b, Q_b\} = 0 \Rightarrow Q_b^2 = 0, \\ s_{ab} Q_{ab} &= -i \{Q_{ab}, Q_{ab}\} = 0 \Rightarrow Q_{ab}^2 = 0, \\ s_b Q_{ab} &= -i \{Q_{ab}, Q_b\} = 0 \Rightarrow Q_{ab} Q_b + Q_b Q_{ab} = 0, \\ s_{ab} Q_b &= -i \{Q_b, Q_{ab}\} = 0 \Rightarrow Q_b Q_{ab} + Q_{ab} Q_b = 0. \end{aligned} \quad (43)$$

We point out that in proving the anticommutativity property ($Q_b Q_{ab} + Q_{ab} Q_b = 0$) of the (anti-)BRST charges we have used the CF conditions (32). For the sake of brevity, one can check

$$\begin{aligned}
s_b Q_{ab} &= -i \int d^2x \left[\bar{B} \cdot \partial^0 (B + \bar{B} + igC \times \bar{C}) \right] \\
&+ \int d^2x \left[g \left((B + \bar{B} + igC \times \bar{C}) \times \beta \right) \cdot D^0 \beta - g D^0 \left((B + \bar{B} + igC \times \bar{C}) \times \beta \right) \cdot \bar{\beta} \right] \\
&- i \int d^2x \left[(R + \bar{R} + igC \times \bar{\beta} + ig\bar{C} \times \beta) \cdot D^0 (R + igC \times \bar{\beta}) \right], \\
\\
s_{ab} Q_b &= i \int d^2x \left[B \cdot \partial^0 (B + \bar{B} + igC \times \bar{C}) \right] \\
&- \int d^2x \left[g \left((B + \bar{B} + igC \times \bar{C}) \times \bar{\beta} \right) \cdot D^0 \beta - g D^0 \left((B + \bar{B} + igC \times \bar{C}) \times \bar{\beta} \right) \cdot \beta \right] \\
&+ i \int d^2x \left[(R + \bar{R} + igC \times \bar{\beta} + ig\bar{C} \times \beta) \cdot D^0 (\bar{R} + ig\bar{C} \times \beta) \right]. \tag{44}
\end{aligned}$$

It is clear from the above expressions that $s_b Q_{ab} = 0$ and $s_{ab} Q_b = 0$ if and only if CF conditions (32) are satisfied. As a consequence, the (anti-)BRST charges are anticommuting only on the constrained hypersurface defined by the CF conditions (32).

6 Ghost scale symmetry and BRST algebra

The Lagrangian densities (34), in addition to the (anti-)BRST symmetry transformations, also respect the continuous ghost scale symmetry (s_g). These symmetry transformations are given as follows

$$\begin{aligned}
C &\rightarrow e^{+\Omega} C, & \bar{C} &\rightarrow e^{-\Omega} \bar{C}, & \beta &\rightarrow e^{+\Omega} \beta, & \bar{\beta} &\rightarrow e^{-\Omega} \bar{\beta}, \\
(A_\mu, \phi_\mu, \rho, B, \bar{B}, R, \bar{R}) &\rightarrow e^0 (A_\mu, \phi_\mu, \rho, B, \bar{B}, R, \bar{R})
\end{aligned} \tag{45}$$

where Ω is the global scale parameter. The numbers $(\pm 1, 0)$ in the exponential of the above transformations stand for ghost numbers of the corresponding fields. For instance, the ghost fields (C, β) carry ghost number $(+1)$ and anti-ghost fields $(\bar{C}, \bar{\beta})$ have ghost number (-1) . The rest (bosonic) fields have ghost number zero. The infinitesimal version of the above continuous transformation is given by

$$\begin{aligned}
s_g C &= +\Omega C, & s_g \bar{C} &= -\Omega \bar{C}, & s_g \beta &= +\Omega \beta, & s_g \bar{\beta} &= -\Omega \bar{\beta}, \\
s_g (A_\mu, \phi_\mu, \rho, B, \bar{B}, R, \bar{R}) &= 0.
\end{aligned} \tag{46}$$

It is straightforward to check that under the above continuous ghost scale symmetry transformations (46) both the Lagrangian densities remain invariant (i.e. $s_g \mathcal{L}_b = s_g \mathcal{L}_{\bar{b}} = 0$). As a consequence, the existence of ghost scale symmetry leads to the following Noether's

conserved current (J_g^μ) and charge (Q_g):

$$\begin{aligned} J_g^\mu &= i \left[\bar{C} \cdot (D^\mu C) - (\partial^\mu \bar{C}) \cdot C + \bar{\beta} \cdot (D^\mu \beta) - (D^\mu \bar{\beta}) \cdot \beta \right], \\ Q_g &= i \int d^2x \left[\bar{C} \cdot (D^0 C) - (\partial^0 \bar{C}) \cdot C + \bar{\beta} \cdot (D^0 \beta) - (D^0 \bar{\beta}) \cdot \beta \right]. \end{aligned} \quad (47)$$

The conservation law ($\partial_\mu J_g^\mu = 0$) can be proven by exploiting the E-L equations of motion (38). The ghost charge Q_g also turns out to be the generator of the ghost scale symmetry transformations (46). For instance, one can check that $s_g C = -i[C, \Omega Q_g] = +\Omega C$.

The above ghost charge Q_g together with the nilpotent (anti-)BRST charges $Q_{(a)b}$ obey a standard BRST algebra. In operator form, this algebra can be given as follow

$$\begin{aligned} Q_b^2 &= 0, & Q_{ab}^2 &= 0, & \{Q_b, Q_{ab}\} &= Q_b Q_{ab} + Q_{ab} Q_b = 0, \\ i[Q_g, Q_b] &= +Q_b, & i[Q_g, Q_{ab}] &= -Q_{ab}, & Q_g^2 &\neq 0. \end{aligned} \quad (48)$$

Let us consider a state $|\psi\rangle_n$, in the quantum Hilbert space of states, such that the ghost number of the state is defined in the following manner

$$iQ_g |\psi\rangle_n = n |\psi\rangle_n, \quad (49)$$

where n is the ghost number of the state $|\psi\rangle_n$. Now, it is easy to check, with the help of above algebra (48), that following relationships holds

$$\begin{aligned} iQ_g Q_b |\psi\rangle_n &= (n+1) Q_b |\psi\rangle_n, \\ iQ_g Q_{ab} |\psi\rangle_n &= (n-1) Q_{ab} |\psi\rangle_n, \end{aligned} \quad (50)$$

which shows that the BRST charge Q_b increases the ghost number by one unit when it operates on a quantum state whereas the anti-BRST charge Q_{ab} decreases it by one unit. In other words, we can say that the (anti-)BRST charge carry the ghost numbers (∓ 1), respectively. A careful look at the expressions of the (anti-)BRST and ghost charges, where the ghost numbers of the fields are concerned, also reveal the same observations.

7 Role of auxiliary field: A bird's-eye view

In this section we provide a brief synopsis about few striking similarities and some glaring differences among the 3D non-Abelian JP model, 4D topologically massive non-Abelian 2-form gauge theory [39, 40] and the 4D modified gauge invariant Proca theory in the realm of well-known Stückelberg formalism (see, e.g. [41] for details).

7.1 Jackiw-Pi model

It is interesting to note that, if we make the following substitution

$$\phi_\mu \longrightarrow \phi_\mu + D_\mu \rho, \quad (51)$$

in our starting Lagrangian density (1), the 2-form $G_{\mu\nu}$ and mass term re-defined as

$$\begin{aligned} G_{\mu\nu} &\longrightarrow G_{\mu\nu} - g(F_{\mu\nu} \times \rho), \\ \frac{m}{2} \varepsilon^{\mu\nu\eta} F_{\mu\nu} \cdot \phi_\eta &\longrightarrow \frac{m}{2} \varepsilon^{\mu\nu\eta} F_{\mu\nu} \cdot \phi_\eta + \partial_\eta \left[\frac{m}{2} \varepsilon^{\mu\nu\eta} F_{\mu\nu} \cdot \rho \right] - \frac{m}{2} \varepsilon^{\mu\nu\eta} (D_\eta F_{\mu\nu}) \cdot \rho. \end{aligned} \quad (52)$$

In the above, the term $\frac{m}{2} \varepsilon^{\mu\nu\eta} (D_\eta F_{\mu\nu}) \cdot \rho$ is zero due to the validity of the well-known Bianchi identity ($D_\mu F_{\nu\eta} + D_\nu F_{\eta\mu} + D_\eta F_{\mu\nu} = 0$). Therefore, the mass term remains invariant, modulo a total spacetime derivative, under the re-definition (51). As a consequence, the modified Lagrangian density, modulo a total spacetime derivative, is given by

$$\tilde{\mathcal{L}}_0 = -\frac{1}{4} F_{\mu\nu} \cdot F^{\mu\nu} - \frac{1}{4} G_{\mu\nu} \cdot G^{\mu\nu} + \frac{m}{2} \varepsilon^{\mu\nu\eta} F_{\mu\nu} \cdot \phi_\eta. \quad (53)$$

It is clear that the auxiliary field ρ is completely eliminated from the above Lagrangian density. We point out that, even though, Lagrangian density (53) respects the YM gauge transformations (2) but it fails to respect the NYM gauge transformations (3). The similar observation can also be seen in the case of 4D topologically massive non-Abelian 2-form gauge theory as well as in the 4D modified gauge invariant version of Proca theory.

7.2 4D massive non-Abelian 2-form gauge theory

The Lagrangian density for the 4D massive non-Abelian 2-form gauge theory is given by (see, for details [11, 39, 40])

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} \cdot F^{\mu\nu} + \frac{1}{12} H_{\mu\nu\eta} \cdot H^{\mu\nu\eta} + \frac{m}{4} \varepsilon^{\mu\nu\eta\kappa} B_{\mu\nu} \cdot F_{\eta\kappa}, \quad (54)$$

where 3-form $H_{\mu\nu\eta} = D_\mu B_{\nu\eta} + D_\nu B_{\eta\mu} + D_\eta B_{\mu\nu} + g(F_{\mu\nu} \times K_\eta) + g(F_{\nu\eta} \times K_\mu) + g(F_{\eta\mu} \times K_\nu)$ is the field strength tensor corresponding to the 2-form gauge field $B_{\mu\nu}$ and the 2-form field strength tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - g(A_\mu \times A_\nu)$ corresponds to the 1-form gauge field A_μ . The coupling constant is represented by g and D_μ is the covariant derivative. The auxiliary field K_μ is the compensating field. This Lagrangian density respects the two types of gauge transformations – the scalar gauge transformation ($\tilde{\delta}_1$) and vector gauge transformation ($\tilde{\delta}_2$), namely; [11, 39, 40]

$$\begin{aligned} \tilde{\delta}_1 A_\mu &= D_\mu \Omega, & \tilde{\delta}_1 B_{\mu\nu} &= -g(B_{\mu\nu} \times \Omega), & \tilde{\delta}_1 K_\mu &= -g(K_\mu \times \Omega), \\ \tilde{\delta}_2 A_\mu &= 0, & \tilde{\delta}_2 B_{\mu\nu} &= -(D_\mu \Lambda_\nu - D_\nu \Lambda_\mu), & \tilde{\delta}_2 K_\mu &= -\Lambda_\mu, \end{aligned} \quad (55)$$

where $\Omega(x)$ and $\Lambda_\mu(x)$ are the local scalar and vector gauge parameters, respectively. We note that if we re-define the $B_{\mu\nu}$ field as

$$B_{\mu\nu} \longrightarrow B_{\mu\nu} + (D_\mu K_\nu - D_\nu K_\mu), \quad (56)$$

the 3-form field strength tensor $H_{\mu\nu\eta}$ and the mass term modify as follows

$$\begin{aligned} H_{\mu\nu\eta} &\longrightarrow \tilde{H}_{\mu\nu\eta} = D_\mu B_{\nu\eta} + D_\nu B_{\eta\mu} + D_\eta B_{\mu\nu}, \\ \frac{m}{4} \varepsilon^{\mu\nu\eta\kappa} B_{\mu\nu} \cdot F_{\eta\kappa} &\longrightarrow \frac{m}{4} \varepsilon^{\mu\nu\eta\kappa} B_{\mu\nu} \cdot F_{\eta\kappa} + \partial_\mu \left[\frac{m}{2} \varepsilon^{\mu\nu\eta\kappa} K_\nu \cdot F_{\eta\kappa} \right] \\ &\quad - \frac{m}{2} \varepsilon^{\mu\nu\eta\kappa} K_\nu \cdot (D_\mu F_{\eta\kappa}). \end{aligned} \quad (57)$$

and the compensating auxiliary vector field K_μ disappears from the Lagrangian density (54). Furthermore, the mass term $\frac{m}{4} \varepsilon^{\mu\nu\eta\kappa} B_{\mu\nu} \cdot F_{\eta\kappa}$ remains intact modulo a total spacetime derivative. Thus, the modified Lagrangian density can be given in the following manner (modulo a total spacetime derivative)

$$\tilde{\mathcal{L}} = -\frac{1}{4} F_{\mu\nu} \cdot F^{\mu\nu} + \frac{1}{12} \tilde{H}_{\mu\nu\eta} \cdot \tilde{H}^{\mu\nu\eta} + \frac{m}{4} \varepsilon^{\mu\nu\eta\kappa} B_{\mu\nu} \cdot F_{\eta\kappa}. \quad (58)$$

Clearly, the above Lagrangian density is no longer invariant under the vector gauge transformation even though it respects the scalar gauge transformations [cf. (55)].

It is clear from the above discussions that both the above models (i.e. JP model and 4D massive non-Abelian 2-form gauge theory) are very similar to each other in the sense that under the re-definitions of the fields ϕ_μ and $B_{\mu\nu}$ the auxiliary fields ρ and K_μ are eliminated from their respective models. As a result, the modified Lagrangian densities (53) and (58) do not respect the symmetry transformations (δ_2) and $(\tilde{\delta}_2)$, respectively. Thus, the auxiliary fields ρ and K_μ are required in their respective models so that these models respect both the gauge symmetry transformations [cf. (2), (3) and (55)].

7.3 Modified version of Abelian Proca theory

The above key observations can also be seen in the case of modified gauge invariant Abelian Proca theory. The gauge invariant Lagrangian density of this model is as follows [41]

$$\mathcal{L}_s = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{m^2}{2} A_\mu A^\mu + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + m A_\mu \partial^\mu \phi, \quad (59)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength tensor corresponding to A_μ , ϕ is the Stückelberg field and m represents the mass of the photon field A_μ . Under the following local gauge transformations

$$\delta_{(gt)} A_\mu = \partial_\mu \chi(x), \quad \delta_{(gt)} \phi = -m \chi(x), \quad (60)$$

the Lagrangian density (59) remains invariant. Here $\chi(x)$ is the local gauge transformation parameter. It can be checked that under the following re-definition

$$A_\mu \longrightarrow A_\mu - \frac{1}{m} \partial_\mu \phi, \quad (61)$$

the Stückelberg field ϕ completely disappears from the Lagrangian density (59). As a consequence, the resulting Lagrangian density does not respect the above gauge transformations (60).

The above observation is very similar to the JP model and the massive non-Abelian 2-form gauge theory. As a consequence, the field ρ (in JP model) and K_μ (in massive non-Abelian 2-form theory) are like the Stückelberg field. However, the key difference is that these Stückelberg like fields (i.e. ρ and K_μ) are auxiliary fields in their respective models whereas, in the modified gauge invariant Proca theory, the Stückelberg field ϕ is dynamical in nature.

8 Conclusions

In our present investigation, we have derived the off-shell nilpotent and absolutely anti-commuting (anti-)BRST symmetry transformations corresponding to the combined YM and NYM symmetries of the JP model. For this purpose, we have utilized the power and strength of augmented superfield approach. The derivation of proper (anti-)BRST symmetries for the auxiliary field ρ is one of the main findings of our present endeavor. These (anti-)BRST symmetry transformations corresponding to the auxiliary field ρ can neither be generated from the conserved (anti-)BRST charges nor deduced by the requirement of nilpotency and/or absolute anticommutativity of the (anti-)BRST symmetry transformations.

One of the main features of the superfield formalism is the derivation of CF conditions which, in turn, ensure the absolutely anticommutativity of (anti-) BRST symmetry transformations. The CF conditions, a hallmark of any non-Abelian 1-form gauge theories [23], appear naturally within the framework of superfield formalism and also have connections with gerbes [33]. In our present case of combined YM and NYM symmetries of JP model, there exist *two* CF conditions (cf. (32)). This is in contrast to the YM symmetries case where there exist only *one* CF condition [21] and in NYM symmetries case, *no* CF condition was observed [22]. Moreover, these CF conditions have played a central role in the derivation of coupled Lagrangian densities (cf. Section 4).

Furthermore, we have obtained a set of coupled Lagrangian densities which respect the above mentioned (anti-)BRST symmetry transformations. The ghost sector of these coupled Lagrangian densities is also endowed with another continuous symmetry - the ghost symmetry. We have exploited this symmetry to derive the conserved ghost charge. Moreover, we have pointed out the standard BRST algebra obeyed by all the conserved charges of the underlying theory.

At the end, we have provided a bird's-eye view on the role of auxiliary field in the context of various massive models. For this purpose, we have taken three different cases of 3D JP model, 4D massive non-Abelian 2-form gauge theory and the 4D modified version of Abelian Proca theory. We have shown that the field ρ (in JP model) and K_μ (in massive non-Abelian 2-form theory) are like Stückelberg field (ϕ) of Abelian Proca model. However, ρ and K_μ are auxiliary fields whereas ϕ is dynamical, in their respective models. Finally, we capture the (anti-)BRST invariance of the coupled Lagrangian densities (cf. (34)), nilpotency and absolute anticommutativity of (anti-)BRST charges (cf. (41)) within the framework of superfield approach.

Acknowledgments

The research work of SG is supported by Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) grant 151112/2014-2.

A (Anti-)BRST invariance, nilpotency and anticommutativity: Superfield approach

It is interesting to point out that the super expansions (11), (21) and (28) can be expressed in terms of the translations of the corresponding superfields along the Grassmannian directions of the $(3, 2)$ -dimensional supermanifold, as

$$\begin{aligned} s_b \Psi(x) &= \frac{\partial}{\partial \theta} \tilde{\Psi}^{(h)}(x, \theta, \bar{\theta}) \Big|_{\theta=0}, & s_{ab} \Psi(x) &= \frac{\partial}{\partial \theta} \tilde{\Psi}^{(h)}(x, \theta, \bar{\theta}) \Big|_{\bar{\theta}=0}, \\ s_b s_{ab} \Psi(x) &= \frac{\partial}{\partial \theta} \frac{\partial}{\partial \theta} \tilde{\Psi}^{(h)}(x, \theta, \bar{\theta}), \end{aligned} \quad (62)$$

where $\Psi(x)$ is any generic field of the underlying 3D theory and $\tilde{\Psi}^{(h)}(x, \theta, \bar{\theta})$ is the corresponding superfield obtained after the application of HC. The above expression captures the off-shell nilpotency of the (anti-)BRST symmetries because of the properties of Grassmannian derivatives, i.e. $\partial_\theta^2 = \partial_{\bar{\theta}}^2 = 0$. Moreover, the anticommutativity property of the (anti-)BRST symmetry transformations is also clear from the expansions (11), (21) and (28), in the following manner

$$\left(\frac{\partial}{\partial \theta} \frac{\partial}{\partial \theta} + \frac{\partial}{\partial \bar{\theta}} \frac{\partial}{\partial \bar{\theta}} \right) \tilde{\Psi}^{(h)}(x, \theta, \bar{\theta}) = 0. \quad (63)$$

Thus, the expressions (62) and (63) provide the geometrical interpretations for the (anti-)BRST symmetry transformations in terms of the translational generators $(\partial_\theta, \partial_{\bar{\theta}})$ along the Grassmannian directions of the $(3, 2)$ -dimensional supermanifold.

Furthermore, the nilpotency of the (anti-)BRST charges can also be realized, within the framework of superfield formalism, in the following manner

$$\begin{aligned} Q_b &= \frac{\partial}{\partial \bar{\theta}} \int d^2x \left[B(x) \cdot \tilde{\mathcal{A}}_0^{(h)}(x, \theta, \bar{\theta}) + i \dot{\tilde{\mathcal{F}}}^{(h)}(x, \theta, \bar{\theta}) \cdot \tilde{\mathcal{F}}^{(h)}(x, \theta, \bar{\theta}) \right. \\ &\quad + \left(R(x) + ig \tilde{\mathcal{F}}^{(h)}(x, \theta, \bar{\theta}) \times \tilde{\tilde{\beta}}^{(h)}(x, \theta, \bar{\theta}) \right) \cdot \tilde{\Phi}_0^{(h)}(x, \theta, \bar{\theta}) \\ &\quad \left. + i \tilde{\mathcal{D}}_0 \tilde{\tilde{\beta}}^{(h)}(x, \theta, \bar{\theta}) \cdot \tilde{\beta}^{(h)}(x, \theta, \bar{\theta}) \right] \Big|_{\theta=0} \\ &\equiv \int d^2x \int d\bar{\theta} \left[B(x) \cdot \tilde{\mathcal{A}}_0^{(h)}(x, \theta, \bar{\theta}) + i \dot{\tilde{\mathcal{F}}}^{(h)}(x, \theta, \bar{\theta}) \cdot \tilde{\mathcal{F}}^{(h)}(x, \theta, \bar{\theta}) \right. \\ &\quad + \left(R(x) + ig \tilde{\mathcal{F}}^{(h)}(x, \theta, \bar{\theta}) \times \tilde{\tilde{\beta}}^{(h)}(x, \theta, \bar{\theta}) \right) \cdot \tilde{\Phi}_0^{(h)}(x, \theta, \bar{\theta}) \\ &\quad \left. + i \tilde{\mathcal{D}}_0 \tilde{\tilde{\beta}}^{(h)}(x, \theta, \bar{\theta}) \cdot \tilde{\beta}^{(h)}(x, \theta, \bar{\theta}) \right] \Big|_{\theta=0}. \end{aligned} \quad (64)$$

This, in turn, implies

$$\frac{\partial}{\partial \theta} Q_b \Big|_{\theta=0} = 0 \quad \implies \quad Q_b^2 = 0, \quad (65)$$

because of the nilpotency property of the Grassmannian derivative (i.e. $\partial_\theta^2 = 0$). It is interesting to point out that the nilpotency of above BRST charge (Q_b), when written in ordinary 3D spacetime,

$$Q_b = \int d^2x s_b \left[B(x) \cdot A_0(x) + i \dot{\bar{C}}(x) \cdot C(x) + \left(R(x) + igC(x) \times \bar{\beta}(x) \right) \cdot \phi_0(x) + iD_0\bar{\beta}(x) \cdot \beta(x) \right], \quad (66)$$

is straightforward and encoded in the nilpotency property ($s_b^2 = 0$) of the BRST transformations (s_b). In other words, $s_b Q_b = -i\{Q_b, Q_b\} = 0$ is true due to above mentioned reason. Moreover, using the CF-conditions, there is yet another way to express the above BRST charge where nilpotency is quite clear, as can be seen from the following expression

$$Q_b = i \frac{\partial}{\partial \theta} \frac{\partial}{\partial \bar{\theta}} \int d^2x \left[\tilde{\mathcal{A}}_0^{(h)}(x, \theta, \bar{\theta}) \cdot \tilde{\mathcal{F}}^{(h)}(x, \theta, \bar{\theta}) + \tilde{\Phi}_0^{(h)}(x, \theta, \bar{\theta}) \cdot \tilde{\beta}^{(h)}(x, \theta, \bar{\theta}) \right] \equiv i \int d^2x s_b s_{ab} \left[A_0(x) \cdot C(x) + \phi_0(x) \cdot \beta(x) \right]. \quad (67)$$

This is true only on the constrained surface spanned by CF conditions. Similarly, we can express the anti-BRST charge (Q_{ab}) in the following two different ways:

$$Q_{ab} = -\frac{\partial}{\partial \theta} \int d^2x \left[\bar{B}(x) \cdot \tilde{\mathcal{A}}_0^{(h)}(x, \theta, \bar{\theta}) + i \dot{\tilde{\mathcal{F}}}^{(h)}(x, \theta, \bar{\theta}) \cdot \tilde{\mathcal{F}}^{(h)}(x, \theta, \bar{\theta}) - \left(\bar{R}(x) + ig\tilde{\mathcal{F}}^{(h)}(x, \theta, \bar{\theta}) \times \tilde{\beta}^{(h)}(x, \theta, \bar{\theta}) \right) \cdot \tilde{\Phi}_0^{(h)}(x, \theta, \bar{\theta}) - i\tilde{D}_0\tilde{\beta}^{(h)}(x, \theta, \bar{\theta}) \cdot \tilde{\beta}^{(h)}(x, \theta, \bar{\theta}) \right] \Big|_{\theta=0} \equiv i \frac{\partial}{\partial \theta} \frac{\partial}{\partial \bar{\theta}} \int d^2x \left[\tilde{\mathcal{A}}_0^{(h)}(x, \theta, \bar{\theta}) \cdot \tilde{\mathcal{F}}^{(h)}(x, \theta, \bar{\theta}) + \tilde{\Phi}_0^{(h)}(x, \theta, \bar{\theta}) \cdot \tilde{\beta}^{(h)}(x, \theta, \bar{\theta}) \right]. \quad (68)$$

In the above, the second expression is valid on the constrained hypersurface parametrized by the CF conditions. The nilpotency of anti-BRST charge (i.e. $Q_{ab}^2 = 0$) is assured by the nilpotency ($\partial_\theta^2 = 0$) of the Grassmannian derivative ∂_θ , as described below

$$\frac{\partial}{\partial \theta} Q_{ab} \Big|_{\bar{\theta}=0} = 0 \implies Q_{ab}^2 = 0. \quad (69)$$

In 3D ordinary space, the above expression (68) can be written in the following fashion

$$Q_{ab} = -\int d^2x s_{ab} \left[\bar{B}(x) \cdot A_0(x) + i \dot{\bar{C}}(x) \cdot \bar{C}(x) + \left(\bar{R}(x) + ig\bar{C}(x) \times \beta(x) \right) \cdot \phi_0(x) + iD_0\beta(x) \cdot \bar{\beta}(x) \right] \equiv -i \int s_{ab} s_b \left[A_0(x) \cdot \bar{C}(x) + \phi_0(x) \cdot \bar{\beta}(x) \right]. \quad (70)$$

Here, the nilpotency of the anti-BRST charge lies in the equation $s_{ab} Q_{ab} = -i\{Q_{ab}, Q_{ab}\} = 0$ because of the fact that $s_{ab}^2 = 0$.

Furthermore, in order to prove the (anti-)BRST invariance of the coupled Lagrangian densities, within the framework of superfield formalism, we first generalize our starting Lagrangian density (\mathcal{L}_0) onto the $(3, 2)$ -dimensional supermanifold, as follows

$$\begin{aligned}\mathcal{L}_0 \rightarrow \tilde{\mathcal{L}}_0 &= -\frac{1}{4} \tilde{\mathcal{F}}^{\mu\nu(h)} \cdot \tilde{\mathcal{F}}_{\mu\nu}^{(h)} - \frac{1}{4} \left[\tilde{\mathcal{G}}^{\mu\nu(h)} + g \tilde{\mathcal{F}}^{\mu\nu(h)} \times \tilde{\rho}^{(h)} \right] \cdot \left[\tilde{\mathcal{G}}_{\mu\nu}^{(h)} + g \tilde{\mathcal{F}}_{\mu\nu}^{(h)} \times \tilde{\rho}^{(h)} \right] \\ &+ \frac{m}{2} \varepsilon^{\mu\nu\eta} \tilde{\mathcal{F}}_{\mu\nu}^{(h)} \cdot \tilde{\Phi}_\eta^{(h)}.\end{aligned}\quad (71)$$

This Lagrangian density ($\tilde{\mathcal{L}}_0$) is free from the Grassmannian variables (cf. Section 3, for details). Therefore, the followings are true

$$\left. \frac{\partial}{\partial \theta} \tilde{\mathcal{L}}_0 \right|_{\theta=0} = 0, \quad \left. \frac{\partial}{\partial \bar{\theta}} \tilde{\mathcal{L}}_0 \right|_{\bar{\theta}=0} = 0, \quad (72)$$

which captures the (anti-)BRST invariance of the starting Lagrangian density \mathcal{L}_0 . Similarly, we can also generalize the coupled Lagrangian densities (34) onto the $(3, 2)$ -dimensional supermanifold in the following manner

$$\begin{aligned}\mathcal{L}_{\bar{b}} \rightarrow \tilde{\mathcal{L}}_{\bar{b}} &= \tilde{\mathcal{L}}_0 - \frac{\partial}{\partial \theta} \frac{\partial}{\partial \bar{\theta}} \left[\frac{i}{2} \tilde{\mathcal{A}}_\mu^{(h)} \cdot \tilde{\mathcal{A}}^{\mu(h)} + \tilde{\mathcal{F}}^{(h)} \cdot \tilde{\tilde{\mathcal{F}}}^{(h)} + \frac{i}{2} \tilde{\Phi}_\mu^{(h)} \cdot \tilde{\Phi}^{\mu(h)} + \frac{1}{2} \tilde{\beta}^{(h)} \cdot \tilde{\tilde{\beta}}^{(h)} \right], \\ \mathcal{L}_b \rightarrow \tilde{\mathcal{L}}_b &= \tilde{\mathcal{L}}_0 + \frac{\partial}{\partial \bar{\theta}} \frac{\partial}{\partial \theta} \left[\frac{i}{2} \tilde{\mathcal{A}}_\mu^{(h)} \cdot \tilde{\mathcal{A}}^{\mu(h)} + \tilde{\mathcal{F}}^{(h)} \cdot \tilde{\tilde{\mathcal{F}}}^{(h)} + \frac{i}{2} \tilde{\Phi}_\mu^{(h)} \cdot \tilde{\Phi}^{\mu(h)} + \frac{1}{2} \tilde{\beta}^{(h)} \cdot \tilde{\tilde{\beta}}^{(h)} \right].\end{aligned}\quad (73)$$

Now, the (anti-)BRST invariance of the above coupled Lagrangian densities is straightforward because of the fact ($\partial_\theta^2 = \partial_{\bar{\theta}}^2 = 0$). Thus, we have

$$\left. \frac{\partial}{\partial \theta} \tilde{\mathcal{L}}_{\bar{b}} \right|_{\bar{\theta}=0} = 0, \quad \left. \frac{\partial}{\partial \bar{\theta}} \tilde{\mathcal{L}}_b \right|_{\theta=0} = 0, \quad (74)$$

which imply the (anti-)BRST invariance of the coupled Lagrangian densities within the framework of superfield formalism.

References

- [1] Schwinger, J. S.: Phys. Rev. **125**, 397 (1962)
- [2] Schwinger, J. S.: Phys. Rev. **128**, 2425 (1962)
- [3] Deser, S., Jackiw, R., Templeton, S.: Ann. Phys. **140**, 372 (1982) [Erratum-ibid. **185**, 406 (1988)]
- [4] Deser, S., Jackiw, R., Templeton, S.: Phys. Rev. Lett. **48**, 975 (1982)
- [5] Freedman, D. Z., Townsend, P. K.: Nucl. Phys. B **177**, 282 (1981)
- [6] Allen, T. J., Bowick, M. J., Lahiri, A.: Mod. Phys. Lett. A **6**, 559 (1991)

- [7] Harikumar, E., Lahiri, A., Sivakumar, M.: Phys. Rev. D **63**, 105020 (2001)
- [8] Gupta, S., Malik, R. P.: Eur. Phys. J. C **58**, 517 (2008)
- [9] Gupta, S., Kumar, R., Malik, R. P.: Eur. Phys. J. C **70**, 491 (2010)
- [10] Gupta, S., Kumar, R., Malik, R. P.: Eur. Phys. J. C **65**, 311 (2010)
- [11] Kumar, R., Malik, R. P.: Eur. Phys. J. C **71**, 1710 (2011)
- [12] Krishna, S., Shukla, A., Malik, R. P.: Int. J. Mod. Phys. A **26**, 4419 (2011)
- [13] Malik, R. P.: Int. J. Mod. Phys. A **27**, 1250123 (2012)
- [14] Henneaux, M., Lemes, V. E. R., Sasaki, C. A. G., Sorella, S. P., Ventura, O. S., Vilar, L. C. Q.: Phys. Lett. B **410**, 195 (1997)
- [15] Lahiri, A.: Phys. Rev.D **55**, 5045 (1997)
- [16] Lahiri, A.: Phys. Rev. D **63**, 105002 (2001)
- [17] Jackiw, R., Pi, S.-Y.: Phys. Lett. B **403**, 297 (1997)
- [18] Dayi, O. F.: Mod. Phys. Lett. A **13**, 1969 (1998)
- [19] Del, Cima O. M.: J. Phys. A **44**, 352001 (2011)
- [20] Del, Cima O. M.: Phys. Lett. B **720**, 254 (2011)
- [21] Gupta, S., Kumar, R., Malik, R. P.: Can. J. Phys. **92**, 1033 (2014)
- [22] Gupta, S., Kumar, R.: Mod. Phys. Lett. A **28**, 1350011 (2013)
- [23] Curci, G., Ferrari, R.: Phys. Lett. B **63**, 91 (1976)
- [24] Ojima, I.: Prog. Theor. Phys. **64**, 625 (1980)
- [25] Hwang, S.: Nucl. Phys. B **322**, 107 (1989)
- [26] Hwang, S.: Nucl. Phys. B **231**, 386 (1984)
- [27] Faizal, M.: Found. Phys. **41**, 270 (2011)
- [28] Faizal, M.: Phys. Lett. B **705**, 120 (2011)
- [29] Faizal, M., Khan, M.: Eur. Phys. J. C **71**, 1603 (2011)
- [30] Faizal, M.: Commun. Theor. Phys. **58**, 704 (2012)
- [31] Faizal, M.: Int. J. Theor. Phys. **52**, 392 (2013)
- [32] Metsaev, R. R.: Theor. Math. Phys. **181**(3), 1548 (2014)

- [33] Bonora, L., Malik, R. P.: Phys. Lett. B **655**, 75 (2007)
- [34] Weinberg, S.: The Quantum Theory of Fields: Modern Applications Vol. II (Cambridge University Press, Cambridge, 1996)
- [35] Bonora, L., Tonin, M.: Phys. Lett. B **98**, 48 (1981)
- [36] Bonora, L., Pasti, P., Tonin, M.: Nuovo Cim. A **63**, 353 (1981)
- [37] Thierry-Mieg, J., Baulieu, L.: Nucl. Phys. B **228**, 259 (1983)
- [38] Kumar, R., Gupta, S., Malik, R. P.: Commun. Theor. Phys. **61**, 715 (2014)
- [39] Malik, R. P.: Euro. Phys. Lett. **91**, 51003 (2010)
- [40] Kumar, R., Malik, R. P.: Euro. Phys. Lett. **94**, 11001 (2011)
- [41] Ruegg, H., Ruiz-Altaba, M.: Int. J. Mod. Phys. A **19**, 3265 (2004)